



Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
In Further Mathematics (9FM0)
Paper 3B: Further Statistics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

AL FM Stats 1 1906 Mark Scheme Final

Qu	Scheme	Marks	AO
1(a)	[Let X = no. of prizes Andreia wins] $X \sim B(40, 0.02)$	M1	3.3
	[Require $P(X \geq 3) = 1 - P(X \leq 2)$] = 0.04567... awrt 0.0457	A1	1.1b
(b)	[Let Y = no. of the bar when Barney wins] $Y \sim \text{NegBin}(3, 0.02)$	(2) M1	3.3
	[$P(Y = 40) = \binom{39}{2} \times 0.02^2 \times 0.98^{37} \times 0.02$	M1	3.4
	= 0.0028071... awrt 0.00281	A1	1.1b
(c)	$E(Y) = \frac{3}{0.02} = \mathbf{150}$	(3) B1	1.1b
		(1)	
(6 marks)			
Notes			
(a)	M1 for selecting a suitable model i.e. $B(40, p)$ where p is any probability Written or used, may be implied by a correct ans or 0.037429... from $P(X = 3)$ A1 for awrt 0.0457 (correct answer only 2/2)		
(b)	1 st M1 for selecting a suitable model ($NB(3, 0.02)$) May be implied by a correct expression 2 nd M1 for use of model to form a correct expression		
SC	$p \neq 0.02$ Allow prob of the form $\binom{39}{2} p^3 (1-p)^{37}$ where $0 < p < 1$ scores M0M1		
	A1 for awrt 0.00281 (accept awrt 2.81×10^{-3}) [correct answer with no working scores 3/3]		
(c)	B1 for 150		

Qu	Scheme	Marks	A
2(a)	{Let C = no of calls in a 20 min period} $C \sim \text{Po}(\dots)$	M1	3.3
	80 calls per 4-hour period gives $\frac{20}{3}$ per 20 mins i.e. $C \sim \text{Po}(\frac{20}{3})$	M1	3.4
	$[P(C > 4)] = 1 - P(C \leq 4)$ $= 0.79437\dots$ awrt 0.794	A1	1.1b
(b)	{ X = no. of 5 min periods with no calls } $X \sim B(4, e^{-\frac{5}{3}})$	M1	3.3
	$P(X = 3) = 0.02186125\dots$ awrt 0.0219	A1	1.1b
(c)	P(exactly one call) $e^{-\frac{5}{3}} \times \frac{5}{3}$ or $e^{-5} \times 5$	M1	2.1
	P(exactly one call in each break) = $\left(e^{-\frac{5}{3}} \times \frac{5}{3} \right) \times (e^{-5} \times 5)$	M1	1.1b
	$= 0.0106052\dots$ awrt 0.0106	A1	1.1b
(3)			
(8 marks)			
Notes			
(a)	1 st M1 for selecting a Poisson model – written or used. May be implied by 2 nd M1 or a correct Answer.		
	2 nd M1 for the correct Poisson $\text{Po}(\frac{20}{3})$ or $\text{Po}(6.67)$ or better seen <u>and</u> writing or using $1 - P(C \leq 4)$		
A1	for awrt 0.794 (correct ans with no incorrect working scores 3/3)		
(b)	M1 for selecting a correct model $B(4, 0.189)$ or better (calc: 0.188875...)		
	A1 for using the model to get awrt 0.0219 (correct ans with no incorrect working scores 2/2)		
(c)	1 st M1 for a correct prob of 1 call (expressions in e or values) (allow 0.31479... or awrt 0.315 or 0.033689... or awrt 0.0337)		
	2 nd M1 for a correct probability statement or expression. E.g. $P(S = 1 S \sim \text{Po}(\frac{5}{3})) \times P(T = 1 T \sim \text{Po}(5))$		
SC	e.g. $F \sim \text{Po}(\lambda)$ used in (b) to find $P(F = 0)$		
	Then if we see $Y \sim \text{Po}(3\lambda)$ and statement $P(F = 1) \times P(Y = 1)$ award M0M1		
A1	for awrt 0.0106 (correct ans with no incorrect working scores 3/3)		

Qu	Scheme	Marks	Ans
3.	{ Let X = the number when the spinner is spun} $\mu = \underline{3}$	B1	1.1b
	$[E(X^2) =] 0.3 + 4 \times 0.1 + 9 \times 0.2 + 16 \times 0.1 + 25 \times 0.3 [= 11.6 \text{ or } \frac{58}{5}]$	M1	1.1b
	$\sigma^2 [= 11.6 - 3^2 =] \underline{2.6}$	A1	1.1b
	$\bar{X} \approx \sim N \left("3", \sqrt{\frac{"2.6"}{80}} \right)$	M1	2.1
	$P(\bar{X} > 3.25) = [P(Z > 1.3867\dots) =] 0.0827589\dots$ (calc) awrt <u>0.0828</u>	A1ft	1.1b
		A1	3.4
(6 marks)			

Notes

	<p>B1 for stating or using mean = 3</p> <p>1st M1 for using the given model to attempt $E(X^2)$ with at least 3 correct products seen</p> <p>1st A1 for $\text{Var}(X) = 2.6$ or $\sigma = \sqrt{2.6} = 1.6124\dots$ (awrt 1.61)</p> <p>ALT Use of pgf (B1 when mean = 3 seen) (M1 when correct $G''(t)$ seen with attempt at $G''(1)$)</p> <p>$G(t) = 0.3t + 0.1t^2 + 0.2t^3 + 0.1t^4 + 0.3t^5$</p> <p>$G'(t) = 0.3 + 0.2t + 0.6t^2 + 0.4t^3 + 1.5t^4$</p> <p>$G''(t) = 0.2 + 1.2t + 1.2t^2 + 6t^3$ leading to $G''(1) = 8.6$</p> <p>2nd M1 for use of CLT – must use \bar{X} and normal <u>or</u> sight of $N \left("3", \sqrt{\frac{"2.6"}{80}} \right)$ with any letter</p> <p>2nd A1ft for a correct mean and variance, ft their 3 and their 2.6</p> <p>This M1A1ft may be implied by sight of correct st. dev. used in a standardisation leading to $P(Z > 1.39)$ Must see correct use of Z</p> <p>NB $\frac{2.6}{80} = 0.0325$ and $\sqrt{\frac{2.6}{80}} = 0.18027\dots$ so allow e.g. $N(3, \text{awrt } (0.180)^2)$</p> <p>3rd A1 for using the normal model to find probability awrt 0.0828</p> <p>ALT Use of $\sum X$ (If see clear attempt at $P(\sum X > 260)$ condone $P(\sum X > 260.5)$ then:</p> <p>2nd M1 for $\sum X \sim N(\dots)$ <u>or</u> any letter $\sim N("240", \sqrt{"2.6" \times 80}^2)$</p> <p>2nd A1ft for mean = "3" $\times 80 = 240$ <u>and</u> variance = "2.6" $\times 80 = 208$</p> <p>May see $P(\sum X > 260.5) = 0.077597\dots$ but it will only score 2nd M1 2nd A1ft and 3rd A0</p>		
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Qu	Scheme	Marks	AO						
4(a)	[T = no. of oak trees in a square] $T \sim \text{Binomial}$	M1	3.3						
Qu	Scheme $T \sim B(6, p)$	A1	Marks 1.1b						
5(a)	$H_0: \lambda = 2.5$ (or $\mu = 7.5$) $H_1: \lambda \neq 2.5$ (or $\mu \neq 7.5$)	B1(2)	2.5						
(b)	Expected frequency for 6 is less than 5 so pool up $E_5 = 13.08$ $P(X \leq 2) = 0.0203$ (calc: $0.020256 \dots$) or $P(X \leq 3) = 0.0591$ $P(X \leq 13) = 0.9784$ so $P(X \geq 14) = 0.0216$ (calc: $0.0215646 \dots$) = 8.313 <table border="1" style="display: inline-table; margin: 5px;"> <tr> <td>$\frac{O_i}{E_i}$</td> <td>4.521</td> <td>29.617</td> <td>21.805</td> <td>7.599</td> <td>24.77</td> </tr> </table> Giving Critical region of $X \leq 2$ or $X \geq 14$ p needed estimating ($p = 0.5$) so $\nu = 5 - 2 = 3$; cv 7.815 Significant result, so Liam's model is not suitable	$\frac{O_i}{E_i}$	4.521	29.617	21.805	7.599	24.77	M1 M1 M1M1	2.1 3.3 1.1b x2
$\frac{O_i}{E_i}$	4.521	29.617	21.805	7.599	24.77				
(b)	[No. of oak trees in a square] $R \sim \text{Po}(3.3)$ awrt 0.0419 (calc: $0.041821366 \dots$) awrt 0.0418	M1 B1ft	3.3 1.2						
(c)	Correct expression for s or t using Poisson [Let $M =$ no of 3-month periods with significant result] $M \sim B(8, "0.0419")$	M1 A1, A1	(3) 1.1b x2						
(d)	H_0 (Poisson) is a good fit (for no. of oak trees per square) H_1 : Poisson is not a good fit (for no. of oak trees per square) $= 0.04153 \dots$ (calc: $0.041394 \dots$) [0.04139 ~ 0.04154]	M1 B1 A1 (3)	3.3 1.1b 1.1b						
(e)	No pooling needed so degrees of freedom is $6 - 2 = 4$	B1	(3) 1b						
(d)	Critical value is 9.488 (accept 9.49) $Y \sim \text{Po}(6.3)$	B1	M1 1.1b 3						
(f)	Not significant so $P(X \leq 3)$ (or $P(X \leq 2)$) is suitable $[= 0.9945147 \dots - 0.049846 \dots]$	B1	M1 2.2b 4						
(f)	Poisson model has 0.945 fit so suggests that oak trees occur <u>random</u> Or binomial suggests deliberately planted or cultivated Therefore the forest is likely to be wild not cultivated	B1	A1 2.2b 1.1b						
	Notes		(2)						
(a)	B1 for both hypotheses in terms of λ or μ (either way around)		(19 marks)						
(a)	1 st M1 for selecting the correct Po model. Notes for use of Po(7.5) may be implied by 2 nd M1								
(a)	2 nd M1 for choosing binomial. A1 for $B(6, p)$ can be in words and allow $B(6, 0.55)$ (2sf or better)								
(b)	1 st M1 for pooling last 2 classes (Allow 13.08 but accept 13.1) $CR \leq 2$ or set notation but not $P(X \leq 2)$ 2 nd M1 for at least 3 correct values or expressions. Either row to at least 2 sf 1 st A1 for a fully correct CR. Can have $X \leq 3$ and $X \geq 14$ or vice								
(b)	1 st B1 for 3 degrees of freedom 2 nd B1ft for critical value of 7.815 (e.g. $\nu = 4$ use 9.488) 3 rd M1 for awrt 0.0419 or awrt 0.0418 or fit a correct conclusion (non-contextual ignore any contradictory contextual comments or fit addition of their two probs provided both are $0 < \text{prob} < 0.025$ (awrt 3sf)								
(c)	This mark can be implied by a fully correct solution ending with correct contextual conclusion 1 st A1 for selecting a correct binomial model. fit their answer to part (b) 2 nd M1 for correct conclusion in context with all other marks scored 2 nd M1 for a correct probability statement of $1 - P(M \leq 1)$ dep on a binomial selected								
(c)	1 st A1 for answer in range [0.04139, 0.04154] dep on use of $B(8, "0.0419")$ or better 2 nd M1 for use of the model with an expression or correct value for s or t								
(d)	1 st M1 for selecting a $Po(6.3)$ model 2 nd M1 for a correct probability statement using their Poisson model and their CR in (a) which may have just one tail								
(d)	B1 for correct hypotheses must mention Poisson: use of $Po(3.3)$ is B0 A1 for awrt 0.945								
(e)	1 st B1 for correct degrees of freedom $\nu = 4$ only 2 nd B1 for selecting correct critical value (9.488 only) 3 rd B1 for <u>not significant</u> conclusion based on 8.749 vs their cv (condone use of $Po(3.3)$ here)								
(f)	1 st B1 for choosing Poisson as better <u>or</u> stating Poisson implies wild <u>or</u> bino'l implies cultivated 2 nd B1 (dep on rejecting bin and accepting Poisson) by clearly stating woodland is wild If the tests give the same results then 2 nd B0 automatically								

Qu	Scheme	Marks	AO
6 (a)	$G(1) = 1 \Rightarrow k \ln 2 = 1$ so $k = \frac{1}{\ln 2}$		
7 (a)(i)	$[B \sim \text{Geo}(\frac{1}{3})]$ $P(B=4) = (\frac{2}{3})^3 \times \frac{1}{3}$	M1	(3) 3
(b)	$\left\{ G(t) = \frac{1}{\ln 2} [\ln 2 - \ln(2-t)] \right\} \Rightarrow \frac{d}{dt} G(t) = \frac{1}{\ln 2} \left[\frac{1}{2-t} \right]$ or $\frac{1}{\ln 2} (2-t)^{-1}$	A1 M1	1.1b 2.1
(ii)	$P(B \leq 5) = 1 - P(B > 5)$ or $1 - (\frac{2}{3})^5$	M1	2.1
	$[E(X) =] G'(1) = \frac{1}{\ln 2} = \frac{211}{243}$	A1 A1	1.1b 1.1b
(b)	$E(B^2) = \frac{\text{Var}(B) + [E(B)]^2}{\ln 2} = \frac{1}{\ln 2} \left[\frac{1}{(2-t)^2} \right]$	M1	(4) 2.1 2.1
	From formula booklet: $E(B) = \frac{1}{\ln 2} = 3$ and $\text{Var}(B) = \frac{1 - \frac{1}{3}}{\ln 2} = 6$	A1	1.1b
	$\text{Var}(X) = G''(1) + G'(1) - [G'(1)]^2 = \frac{1}{\ln 2} + \frac{1}{\ln 2} - \left(\frac{1}{\ln 2} \right)^2$	M1	2.1
	So $E(B^2) = 6 + 9 = \underline{15}$	A1	1.1b
(c)	[Let R = no. of the spin when it first lands on red] $X = R \sim \text{Geo}(\frac{2}{3} \ln 2)$	M1	(3) 3.3 1.1b
(c)	Requires $E(e^X) = \sum_{x=1}^{\infty} e^x \left(\frac{1}{3}\right)^{x-1} \frac{2}{3}$ by Maclaurin need $G'''(0)$	M1	(7) 3.1 3.1a
	$G'''(t) = \frac{1}{\ln 2} \frac{2}{(2-t)^3} = \frac{2e}{3} \sum_{x=1}^{\infty} \left(\frac{e}{3}\right)^{x-1}$	M1	A1ft 2.1 1.1b
	$P(X=3) = \frac{G'''(0)}{3!} = \frac{2e}{3} \times \frac{1}{1 - \frac{e}{3}}$ or $\frac{2e}{3-e}$	A1	M1 1.1b 3.2a
	$E(e^X) = 19.297... \{ > 15 = E(B^2) \}$ so $\frac{1}{\ln 2} = 0.0601122... \text{ awrt } \underline{0.0601}$	A1	1.1b 2.2a
	Tamara should choose red since it has the higher expected score		(4) 2.2a
			(5) 12 marks
	Notes		(12 marks)
(a)	B1 for finding k (must be exact)		Notes
(a)(i)	M1 for selecting the correct model i.e. $\text{Geo}(p)$ (May be implied by a correct expression)		
(b)	1 st M1 for an attempt to differentiate $G(t)$ or $A(2-t)^{-1}$ (o.e.)		
(ii)	M1 A1 for a correct first derivative (condense k or use of $\frac{1}{2-t}$ = awrt 1.44)		
	1 st A1 for for correct strategy to use the geometric model to find a correct expression		
	2 nd A1 for for correct strategy to use the geometric model to find a correct expression		
	2 nd M1 for attempting second derivative (ft their $G'(t)$)		
(b)	M1 A1 for for a correct strategy to find $E(B^2)$ (allow $G''(1)$ or $G'(1)$ = awrt 1.44)		
	B1 M1 for use of the correct formula to find $E(B)$, $\text{Var}(B)$ and $\text{Var}(X)$ (the correct formula)		
	A1 for 15		
	4 th A1 for $\frac{1}{\ln 2} \left(2 - \frac{1}{\ln 2} \right)$ o.e. but must simplify i.e. collect like terms		
SC	Formula for $E(B^2)$ Allow M1B1A0 for $E(B^2) = \frac{2-p}{\ln 2}$ (o.e.)		
	[Mark final answer – penalise incorrect log work etc]		
	NB 0.8040211.. is A0 unless exact answer seen		
(c)	1 st M1 for choosing a suitable geometric model (sight of $\text{Geo}(\frac{2}{3})$ or at least 3 correct probabilities)		
(c)	1 st M1 for a suitable strategy to solve the problem (finding link with Maclaurin)		
	2 nd M1 for realising the need for appropriate expected value and using $E(g(X))$ (Need sum and $f(x)$)		
	Need mention of coefficient of t^3 and $G'''(t)$ or $G'''(0)$ (condense $G'(t)$)		
	NB simply finding $e^{19.3} = e^{19.3}$ = awrt 4.48 is M0 and probably no more marks.		
	1 st A1ft for 3 rd derivative, ft their 2 nd derivative in (b) (provided $G''(t)$ not const)		
	3 rd M1 for a suitable strategy to turn the expression into a sum that can be found		
	Correct $G'''(t)$ or $G'''(0)$ scores 1 st M1 1 st A1ft		
	1 st A1 for correct use of sum to infinity of geometric series		
	2 nd M1 for translating Maclaurin to probability (a correct expression)		
	2 nd M1 for interpreting the outcome of the calculations in terms of a solution to the problem must		
	2 nd A1 choose red and see the awrt 19.3 (and allow ft of their $E(B^2) < 19$)		
ALT	Log series 1 st M1 attempt to write $G(t)$ in suitable form as far as: $k[\ln 2 - \ln(2[1 - \frac{t}{2}])]$		
	1 st A1 reaching $-k \ln(1 - \frac{t}{2})$		
	2 nd M1 use of $-\ln(1-x)$ series (some correct substitution) NB $G(t) = \frac{1}{\ln 2} \left(\frac{t}{2} + \frac{t^2}{8} + \frac{t^3}{24} + \dots \right)$		

